Computational Models for Parallel Accelerators

HW/Theory Reading Group

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Motivation

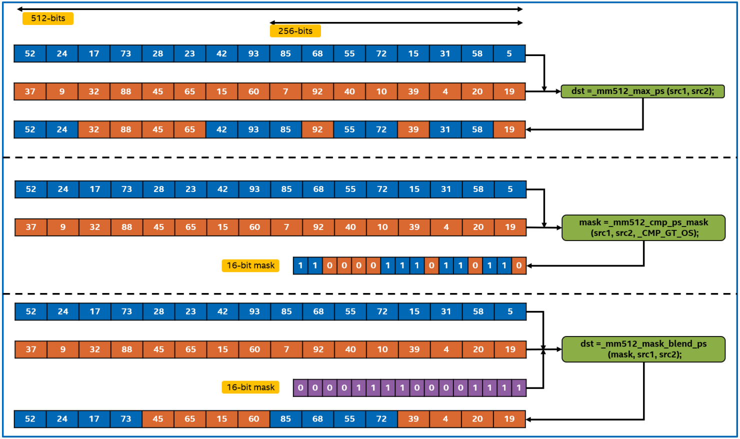
"Fundamental" operations in traditional models of computation: bitwise operations (AND, OR, XOR, NOT, etc.) or arithmetic operations ($+$, $-$, $\times$, $/$).

Arithmetic circuits, Turing machines...

But in real chips have **parallelism**, and **only in restricted shapes**

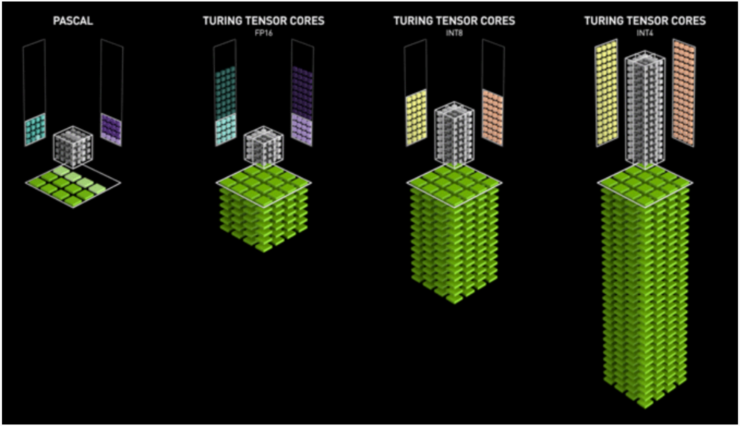
Motivation

Vector parallelism (image source: Intel AVX-512 docs):



Motivation

NVIDIA TCU:



Motivation

How to **model** these forms of “restricted” parallelism?

Treat parallelism as constant factor: “the ostrich approach” - bury your head in the sand and just ignore hardware factors

Multi-processor models (e.g. NC): don’t account for far more limited forms of parallelism for these architectures, communication costs, etc.

Solution: new computational models with new primitives

Tensor Computational Units [1]

Fundamental operation: matrix multiplication

Specifically: a (m, l)-TCU model, a $\sqrt{m} \times \sqrt{m} \times \sqrt{m}$ matrix multiplication in $O(m + l)$ time.

Intuition: $\sqrt{m} \times \sqrt{m}$ systolic array with l latency cost per instruction.

Asymmetric matmuls $(n \times \sqrt{m} \times \sqrt{m})$ can be performed in $O(n\sqrt{m} + l)$ time.

TCUs and External-Memory Models

Very related to models we've looked at!

Theorem: Suppose some problem P with two input matrices and one output matrices has a lower bound F_p on communication complexity on a system with memory $M = 3m + O(1)$ with constant block size. Then any algorithm for P in the *weak TCU model* (i.e. no ability to take advantage of "long" matrices) requires at least $\Omega(F_p)$ time.

Proof: each call to tensor unit is equivalent to loading two $\sqrt{m} \times \sqrt{m}$ matrices into memory and computing on the output.

Strassen-like Matmuls on TCUs

Strassen-like matmul splits each matrix into n_0 equally-sized pieces and performs p_0 recursive matmuls on the matrices, for a runtime of $O(n^{w_0 := \log_{n_0} p_0})$. Classical n^3 matmul is $(n_0, p_0) = (4, 8)$, Strassen is $(4, 7)$.

Theorem [1]: exists a (m, l)-TCU algorithm performing $\sqrt{n} \times \sqrt{n} \times \sqrt{n}$ matmul (s.t. $m \geq n_0$) in time

$$T(n) = O\left(\left(\frac{n}{m}\right)^{\omega_0}(m + l)\right)$$

Proof: Perform recursive matmul until small enough to fit inside $\sqrt{m}$. Theorem comes immediately from solving the recurrence:

$$T_n = \begin{cases} O\left(\frac{n^{3/2}}{m^{1/2}} + \frac{n}{m} l\right) & \text{if } m \leq n \leq mn_0 \\ p_0 T(n/n_0) + O(n) & \text{otherwise} \end{cases}$$

Cooley-Tukey: break an FFT of size $n = n_1 n_2$ into (a) n_1 subproblems of size n_2, (b) multiplication by roots of unity ($O(n)$ size operation), and (c) n_2 operations of size n_1.

Theorem: DFT of a vector with n entries on (m, l)-TCU costs $O((n + l) \log_m n)$.

Proof: set $n_1 = \sqrt{m}$, $n_2 = n/\sqrt{m}$. Cost of (a): $\sqrt{m}T(n/\sqrt{m})$. Cost of (b): $O(n)$. Cost of (c): a single 'long multiplication', $O(n + l)$

$$T = \begin{cases} \sqrt{m}T(n/\sqrt{m}) + O(n + l) & n > m \\ O(m + l) & n \leq m \end{cases}$$

Solving gives the desired result.

Other Results

Sparse matmul, evenly balanced output sparsity:
$O\left(\sqrt{\frac{n}{\text{nnz}(out)}}\left(\frac{\text{nnz}(out)}{m}\right)^{\omega_0}(m+l)+\text{nnz}(in)\right)$

APSP on n-vertex graph: $O((n^2/m)^{\omega_0}(m+l)\log n)$

Thanks for listening! Questions?

References i

Chowdhury, Rezaul, Francesco Silvestri, and Flavio Vella (2020). "A Computational Model for Tensor Core Units". In: *Proceedings of the 32nd ACM Symposium on Parallelism in Algorithms and Architectures*. New York, NY, USA: Association for Computing Machinery, pp. 519–521. ISBN: 9781450369350. URL: `https://doi.org/10.1145/3350755.3400252`.